TECHNOLOGY

# Wheatstone Bridge Nonlinearity

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# Wheatstone Bridge Nonlinearity

## Introduction

Commercial static strain indicators and signal conditioners vary considerably in their circuit details; and, although most of them are based upon some form of the Wheatstone bridge circuit, the bridge circuit is employed in differing ways in different instruments. Because of the many variations in instrument design, a completely general treatment of instrument nonlinearities is not practicable within the scope of this publication. There is, however, a large class of static strain indicators and signal conditioners with a more-or-less characteristic circuit arrangement (employing the "unbalanced" Wheatstone bridge), and displaying a characteristic nonlinearity. This publication has been prepared to provide a simple means for determining the magnitudes of the nonlinearity errors and for making corrections when necessary. It should be noted that the error and correction relationships given here apply only to instruments having the characteristics defined in the next section. For other strain indicators, the nonlinearity errors, if they exist, will have to be determined by direct calibration or from manufacturers' specifications.

The nonlinearity error occurs because, when strain measurements are made with an "unbalanced" Wheatstone bridge circuit (as described in the next section), there are certain conditions under which the output of the bridge circuit is a nonlinear function of the resistance change(s) producing that output. The error due to the nonlinearity, when present, is ordinarily small, and can usually be ignored when measuring elastic strains in metals. However, the percentage error increases with the magnitude of the strain being measured, and can become quite significant at large strains (for example, the error is about 0.1% at 1000 microstrain, 1% at 10 000 microstrain, and 10% at 100 000 microstrain; or, as a convenient rule of thumb, the error, in percent, is approximately equal to the strain, in percent).

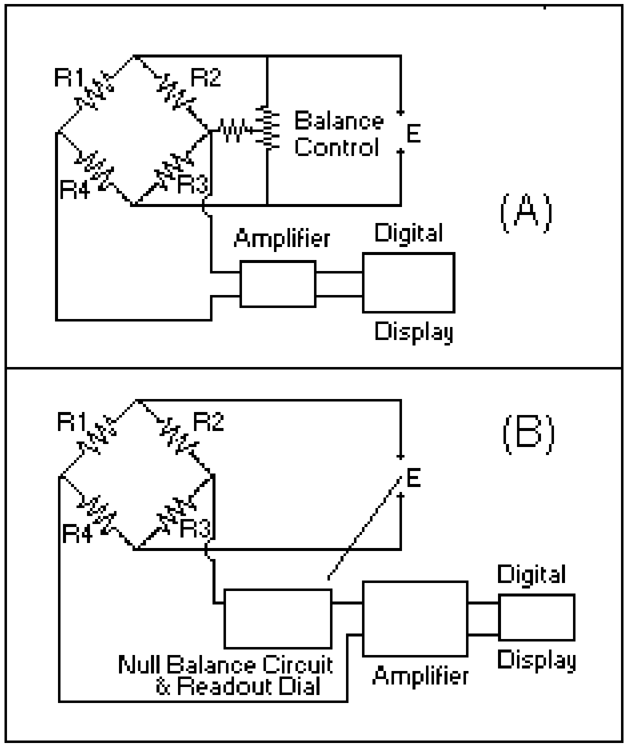
# Wheatstone Bridge Nonlinearity

## The "Unbalanced" Wheatstone Bridge Circuit

Most static strain indicators and signal conditioners for use with resistance strain gages use a form of the Wheatstone bridge circuit in which the bridge arms consist of one to four active gages. The classical Wheatstone bridge arrangement has been used for many years for the accurate measurement of a single unknown resistance; and, in such instruments, the bridge is balanced at the time of measurement by adjusting the resistances of the other arms. The bridge circuit found in most strain indicators, on the other hand, is unbalanced by the varying gage resistance(s) at the time of making the measurement, and is therefore commonly referred to as the "unbalanced" Wheatstone bridge.

The output voltage obtained from the "unbalanced" Wheatstone bridge is a function of the amount of unbalance, and is therefore directly related to the strain applied to the strain gage. However, under certain conditions frequently encountered in actual practice, the bridge output voltage is, as noted earlier, a nonlinear function of the resistance change in the bridge arms; and, when this occurs, the strain readings will be somewhat in error.

Shown below are two of the circuit arrangements most commonly employed in commercial strain indicators and signal conditioners. In circuit (A), the bridge output voltage is amplified and displayed on an indicating instrument, frequently a digital voltmeter. In circuit (B), the bridge output voltage is "nulled" by an equal and opposite voltage injected into the measurement circuit.




In both cases, the nonlinearity errors are identical if the amplifiers have high input impedances, and if the power supplies are of the constant-voltage type. Note also that in both circuits the "balance" control is used only to establish initial bridge balance before the gages are strained, and that the balance controls do not form part of the readout circuit. This type of "balance" circuit is normally provided with a very limited range so as not to cause problems in resolution and setting-stability; and therefore does not greatly influence the nonlinearity errors as described in this publication. To permit a rigorous treatment of the errors without introducing other considerations, it is assumed throughout the following discussion that the "balance" circuit is either completely disconnected, or that the control is left at the midpoint of its range. It is also assumed that the bridge arms are nominally resistively symmetrical about an axis joining the output corners of the bridge; i.e. that:

$$(R_1/R_4)_{nom} = 1 = (R_2/R_3)_{nom}$$

As a result of the circuit arrangements described above, obtaining a reading from the static strain indicator (whether or not the process involves nulling a meter) has no effect on the state of resistive balance within the Wheatstone bridge circuit. Even if the Wheatstone bridge is initially balanced resistively so that $R_1/R_4 = R_2/R_3$, this will no longer be true, in general, when one or more of the strain gages in the bridge arms are strained. Consequently, the Wheatstone bridge is ordinarily operated in a resistively unbalanced state. In this mode of operation, resistance changes in the bridge arms may cause changes in the currents through the arms, depending upon the signs and magnitudes of the resistance changes in all four arms. When current changes occur, the voltage output of the bridge is not proportional to the resistance changes, and thus the output is nonlinear with strain, and the instrument indication is in error.

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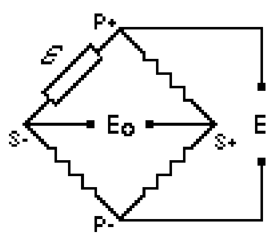
# Wheatstone Bridge Nonlinearity

## Error Magnitudes and Corrections

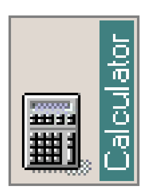
The Bridge Arrangement Table gives, for the class of instruments described in the previous section, the output voltage as a function of the applied strain for a variety of cases representing different strain states and different arrangements of gages on the structural member and within the Wheatstone bridge. While the magnitudes of the nonlinearities are difficult to judge from the table, it can be seen from the column of bridge and strain arrangements that only when the resistance changes are such that the currents in the bridge arms remain constant - that is, when $\Delta R_1 / R_1 + \Delta R_4 / R_4 = 0$ and $\Delta R_2 / R_2 + \Delta R_3 / R_3 = 0$ - is the output a linear function of the strain. The table also includes, for each case, the ratio of the actual strain to the uncorrected strain, permitting correction of indicated strains with these formulas in the nonlinear cases.

### Quarter Bridges

The first case in the accompanying Bridge Arrangement Table is applicable whenever a single active strain gage is used in a quarter-bridge arrangement (as shown below); and occurs very commonly in the practice of strain measurement for experimental stress analysis purposes.




Because of its basic importance, this case will later be used in several numerical examples to demonstrate the procedure for making nonlinearity corrections. The character of the nonlinearity associated with the quarter-bridge arrangement can be illustrated by writing the bridge output equation in the following form:




$$\frac{E_o}{E} = \frac{F\varepsilon \times 10^{-3}}{4}\left(\frac{2}{2 + F\varepsilon \times 10^{-6}}\right)$$ Eq. (507.1)

where:

- $\frac{E_o}{E}$ = dimensionless bridge output, mV/V
- $E_o$ = output voltage, mV
- $E$ = bridge supply voltage, V
- $F$ = gage factor of strain gage
- $\varepsilon$ = actual strain, microstrain

In Eq. (507.1), the term in parentheses represents the nonlinearity. It is evident from the form of the nonlinearity term that its magnitude will be less than unity for tensile strains and greater than unity for compressive strains. And the errors in strain indication due to the nonlinearity will correspond. In other words, indicated tensile strains will be too small and indicated compressive strains too large. For subsequent convenience, the incremental nonlinearity error, or correction, ($\eta$), is defined as the amount which must be added algebraically to the indicated strain to obtain the actual strain. That is,

$$\varepsilon = \hat{\varepsilon} + \eta$$ Eq. (507.2)

where:

- $\varepsilon$ = actual strain causing a resistance change in one arm of the Wheatstone bridge, microstrain
- $\hat{\varepsilon}$ = indicated strain (corresponding to $\varepsilon$) as read from a strain indicator with the specifications given in the Bridge Arrangement Table, microstrain
- $\eta$ = incremental error in indicated strain, microstrain

For the single active gage in a quarter-bridge arrangement, it can be shown that the incremental error (in microstrain) is represented by the following expression:

$$\eta = \frac{F(\hat{\varepsilon})^2 \times 10^{-6}}{2 - F\hat{\varepsilon} \times 10^{-6}}$$ Eq. (507.3)

The correction [which, from Eq. (507.3), always has a positive sign, irrespective of the sign of the indicated strain] is to be added algebraically, to the indicated strain. That is, the magnitude of an indicated tensile strain is always increased by adding the correction, while that of a compressive strain is always reduced.

# Wheatstone Bridge Nonlinearity

## Numerical Examples

Example (1) As a first example, assume that a Wheatstone bridge with a single active gage (quarter-bridge) was initially balanced resistively, after which the gaged test member was loaded until the strain indicator registered 15 000 microstrain in tension. Calculation yields the correction as 230 microstrain at a gage factor of 2.0. The actual strain is thus 15 230 microstrain.

Example (2) It was assumed in the previous example that the Wheatstone bridge was initially in a state of resistive balance. In the practice of experimental stress analysis with strain gages, this may not always be the case. For instance, during the bonding of a strain gage the resistance of the gage may be altered significantly from the manufactured value by poor installation technique. It may also happen that the gage is strained to the plastic range by assembly or preload stresses before subsequent strain measurements are to be made. The initial resistive unbalance, unless it is known to be insignificant, should be measured and properly accounted for in making nonlinearity corrections. When great enough to warrant consideration, the initial unbalance (expressed in strain units) must be added algebraically to any subsequent observed strains so that the nonlinearity correction is based on the total (or net) unbalance of the Wheatstone bridge at any stage in the strain measurement process.

For this example, assume that by interchanging the connections to the active and dummy arms of the Wheatstone bridge, the strain indicator indicates an initial unbalance of -4500 microstrain in an installed strain gage. This is an indicated unbalance, and includes a small nonlinearity error which will be corrected for, in this case, to illustrate the procedure. By calculation, the correction is 20 microstrain, and thus the actual resistive unbalance is -4480 microstrain. After taking this reading (but not resistively balancing the Wheatstone bridge arms), the gaged test object is loaded until the indicated applied strain is -8000 microstrain. The total indicated unbalance in the Wheatstone bridge is then -12 500 microstrain, for which the correction, by calculation, is 155 microstrain. The actual total unbalance is therefore -12 345 microstrain, and the actual applied strain is thus -12 345 - (-4480) = -7865 microstrain.

Example (3) As a final example, consider a case in which the indicated initial unbalance after installing the strain gage was -2500 microstrain. Then the gaged member was installed in a structure with an indicated assembly strain of -45 500 microstrain. After taking this reading, subsequent loading produced an indicated strain change of 3000 microstrain in the tension direction. What corrections should be made to determine the actual tensile strain caused by loading the structure?

Prior to loading the structure, the Wheatstone bridge was unbalanced by an indicated -48 000 microstrain. By calculation, the correction is 2200 microstrain. Thus, the actual unbalance prior to loading was -45 800 microstrain. After loading the structure, the indicated unbalance in the Wheatstone bridge was -48 000 + 3000 = -45 000 microstrain. The correction for this indicated strain (by a second calculation) is 1940 microstrain, and the actual unbalance after loading was -43 060 microstrain. The applied tensile strain due to loading the structure was thus -43 060 - (-45 800) = 2740 microstrain. This example demonstrates that even with relatively modest working strains the nonlinearity error can be very significant (about 10% in this instance) if the Wheatstone bridge is operating far from its resistive balance point.

# Wheatstone Bridge Nonlinearity

## Nonlinearities in Shunt Calibration

The nonlinearity error described in the preceding sections of this publication must always be kept in mind during the shunt calibration of a static strain indicator or signal conditioner. In conventional practice, the strain gage is momentarily shunted by a large resistance, the magnitude of which is selected to produce a decreased resistance in the bridge arm corresponding to a predetermined compressive strain in the gage (at a specified gage factor). When this is done, the strain indicated by the instrument will be in error by the amount calculated from Eq. (507.3) (for compression), and should be corrected accordingly. If the Wheatstone bridge has an initial resistive unbalance, correction for the nonlinearity must be made as demonstrated in the preceding second and third examples, in order to properly account for this condition. Micro Measurements Catalog A-110 includes a tabular list of precision shunt-calibration resistors for simulating different compressive strain magnitudes from 100 microstrain to 10 000 microstrain.

In order to calibrate a strain-indicating instrument for tensile strains, the adjacent inactive arm of the bridge can be shunted by resistances having special magnitudes selected for this purpose. In such cases, the instrument nonlinearity error must be corrected for the tension case.

# Wheatstone Bridge Nonlinearity

## Nonlinearities in Dynamic Strain Measurements

Whenever dynamic strain measurements are made with a Wheatstone bridge circuit, the bridge is always operated in the unbalanced mode. Therefore, the nonlinearities listed in the Bridge Arrangement Table of this publication apply to every such dynamic strain measurement assuming, again, that the bridge is initially balanced resistively. Under these conditions, the error due to the nonlinearity is ordinarily small at typical working strain levels. However, if the bridge is initially unbalanced, the nonlinearity error can be much greater; and, with large initial unbalances, may result in significantly inaccurate strain indications.

# Wheatstone Bridge Nonlinearity

## Summary

The nonlinearity errors occurring in conventional strain gage bridge circuits are normally small enough to ignore when measuring modest strain magnitudes such as those encountered in the elastic range of metals (if the bridge is initially balanced resistively). Large resistive unbalances can, on the other hand, lead to sizable errors in strain indication. The relationships and procedures presented in this publication can be used when necessary to correct for such errors. It also follows that for accurate strain measurements it is imperative to select strain gages with tightly controlled resistance tolerances, and to minimize resistance shifts during gage bonding by carefully following recommended installation techniques.

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# Bridge Output and Nonlinearity Errors

### For Various Bridge/Strain Arrangements in a Uniaxial Stress Field

Including Calculators

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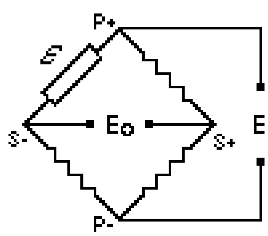
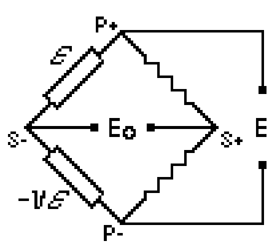
## One Active Gage

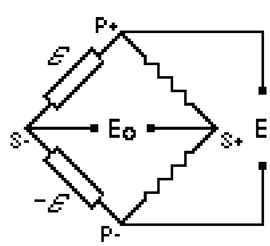



Single tensile or compressive strain. (Nonlinear)

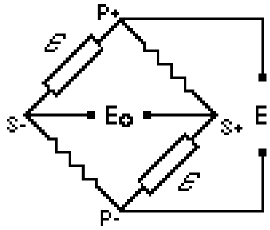
## Two Active Gages




"Poisson" strains in adjacent arms. (Nonlinear)

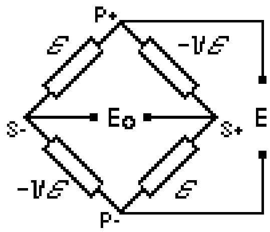



Equal strains of opposite sign in adjacent arms.



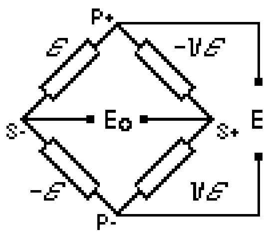

Equal strains of same sign in opposite arms. (Nonlinear)

## Four Active Gages

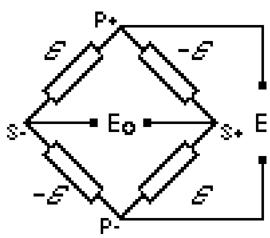



"Poisson" strains of same sign in opposite arms.

(Nonlinear)




"Poisson" strains of opposite sign in adjacent arms.




Equal strains of opposite sign in adjacent arms.

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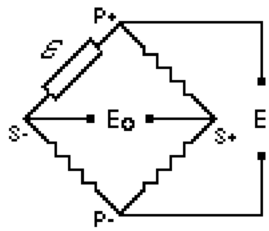
# Wheatstone Bridge Output & Nonlinearity

## One Active Gage




**Description** (Note 1) Single active gage in uniaxial tension or compression.

**Output Equation** (Note 2) $$\frac{E_o}{E} = \frac{F\varepsilon \times 10^{-3}}{4 + 2F\varepsilon \times 10^{-6}} \text{ mV/V}$$

Calculator

(Nonlinear)

**Nonlinearity** $$\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{\varepsilon}{\varepsilon_I} = 1 + \frac{F\hat{\varepsilon} \times 10^{-6}}{2 - F\hat{\varepsilon} \times 10^{-6}}$$

**Actual Strain** $$\varepsilon = \left(\frac{2\varepsilon_I \times 10^{-6}}{2 - F\varepsilon_I \times 10^{-6}}\right) \times 10^6 \text{ microstrain}$$

Calculator

where:

$F$ = gage factor of both gage and instrument

$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction

$\varepsilon_I$ = total strain indicated by the strain gage instrument

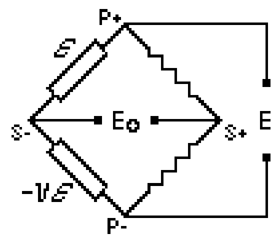
$\hat{\varepsilon}$ = uncorrected axial strain

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Also, $\varepsilon$, $\hat{\varepsilon}$ and $\varepsilon_I$ are expressed in microstrain units.

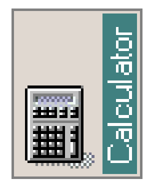
# Wheatstone Bridge Output & Nonlinearity

## Two Active Gages




**Description** (Note 1) Two active gages in uniaxial stress field--one aligned with maximum principal strain, the other with transverse "Poisson" strain.

**Output Equation** (Note 2) $\frac{E_o}{E} = \frac{F\varepsilon\,(1+\nu)\times 10^{-3}}{4+2F\varepsilon\,(1-\nu)\times 10^{-6}}$ mV/V

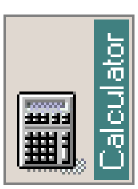



(Nonlinear)

**Nonlinearity**

$$\frac{Actual\ Strain}{Indicated\ Strain} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{\varepsilon(1+\nu)}{\varepsilon_I} = 1 + \frac{F\hat{\varepsilon}\,(1-\nu)\times 10^{-6}}{2 - F\hat{\varepsilon}\,(1-\nu)\times 10^{-6}}$$

**Actual Strain** $\varepsilon = \left(\frac{2\varepsilon_I \times 10^{-6}}{2(1+\nu) - F(1-\nu)\varepsilon_I \times 10^{-6}}\right)\times 10^6$ microstrain




where:

$\nu$ = Poisson's ratio of the strained material

$F$ = gage factor of both gage and instrument

$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction

$\varepsilon_I$ = total strain indicated by the strain gage instrument

$\hat{\varepsilon}$ = uncorrected axial strain

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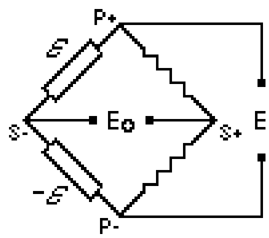
**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.

# Wheatstone Bridge Output & Nonlinearity

## Two Active Gages




**Description** (Note 1) Two active gages with equal and opposite strains -- typical of bending-beam arrangement.

**Output Equation** (Note 2) $\frac{E_o}{E} = \frac{F\varepsilon \times 10^{-3}}{2}$ mV/V

**Nonlinearity Effects** $\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{2\varepsilon}{\varepsilon_I} = 1$ (No nonlinearity errors.)

**Actual Strain** $\varepsilon = \frac{\varepsilon_I}{2}$ microstrain

where:

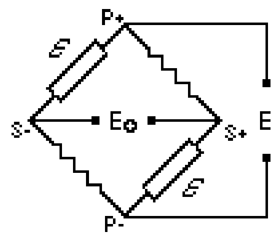
$F$ = gage factor of both gage and instrument
$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction
$\varepsilon_I$ = total strain indicated by the strain gage instrument
$\hat{\varepsilon}$ = uncorrected axial strain

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.

# Wheatstone Bridge Output & Nonlinearity

## Two Active Gages




**Description** (Note 1) Two active gages with equal strains of the same sign -- used on opposite sides of column with low thermal gradient (bending cancellation, for instance.)

**Output Equation** (Note 2) $\frac{E_o}{E} = \frac{F\varepsilon \times 10^{-3}}{2 + F\varepsilon \times 10^{-6}}$ mV/V

(Nonlinear)

**Nonlinearity** $\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{2\varepsilon}{\varepsilon_I} = 1 + \frac{F\hat{\varepsilon} \times 10^{-6}}{2 - F\hat{\varepsilon} \times 10^{-6}}$

**Actual Strain** $\varepsilon = \left( \frac{2\varepsilon_I \times 10^{-6}}{4 - F\varepsilon_I \times 10^{-6}} \right) \times 10^6$

where:

$F$ = gage factor of both gage and instrument

$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction

$\varepsilon_I$ = total strain indicated by the strain gage instrument

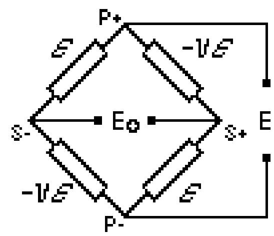
$\hat{\varepsilon}$ = uncorrected axial strain

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.

# Wheatstone Bridge Output & Nonlinearity

## Four Active Gages

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**Description** (Note 1) Four active gages in uniaxial stress field--two aligned with maximum principal strain, the other two with transverse "Poisson" strain (column).

**Output Equation** (Note 2) $$\frac{E_o}{E} = \frac{F\varepsilon\,(1+\nu)\times 10^{-3}}{2+F\varepsilon\,(1-\nu)\times 10^{-6}} \text{ mV/V}$$

(Nonlinear)

**Nonlinearity**

$$\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{2\varepsilon\,(1+\nu)}{\varepsilon_I} = 1 + \frac{F\hat{\varepsilon}\,(1+\nu)\times 10^{-6}}{2 - F\hat{\varepsilon}\,(1+\nu)\times 10^{-6}}$$

**Actual Strain** $$\varepsilon = \left(\frac{2\varepsilon_I\times 10^{-6}}{4(1+\nu) - F\varepsilon_I(1-\nu)\times 10^{-6}}\right)\times 10^{6}$$

where:

$\nu$ = Poisson's ratio of the strained material

$F$ = gage factor of both gage and instrument

$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction

$\varepsilon_I$ = total strain indicated by the strain gage instrument

$\hat{\varepsilon}$ = uncorrected axial strain

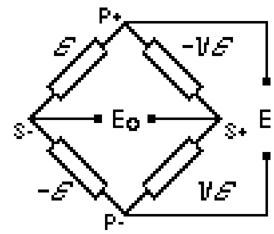
---

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.

# Wheatstone Bridge Output & Nonlinearity

## Four Active Gages




**Description** (Note 1) Four active gages in uniaxial stress field--two aligned with maximum principal strain, the other two with transverse "Poisson" strain (beam).

**Output Equation** (Note 2) $\frac{E_o}{E} = \frac{F\varepsilon(1+\nu) \times 10^{-3}}{2}$ mV/V

**Nonlinearity Effects** $\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{2\varepsilon(1+\nu)}{\varepsilon_I} = 1$ (No nonlinearity errors.)

**Actual Strain** $\varepsilon = \frac{\varepsilon_I}{2(1+\nu)}$

where:

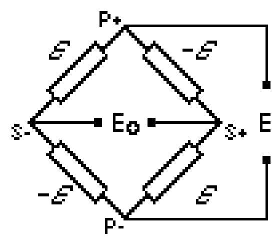
$\nu$ = Poisson's ratio of the strained material
$F$ = gage factor of both gage and instrument
$\varepsilon$ = the actual strain, in uniaxial tension or compression, in the axial direction
$\varepsilon_I$ = total strain indicated by the strain gage instrument
$\hat{\varepsilon}$ = uncorrected axial strain

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.

# Wheatstone Bridge Output & Nonlinearity

## Four Active Gages




**Description** (Note 1) Four active gages with pairs subjected to equal and opposite strains (beam in bending or shaft in torsion).

**Output Equation** (Note 2) $\frac{E_o}{E} = F\varepsilon \times 10^{-3}$ mV/V

**Nonlinearity Effects** $\frac{\text{Actual Strain}}{\text{Indicated Strain}} = \frac{\varepsilon}{\hat{\varepsilon}} = \frac{4\varepsilon}{\varepsilon_I} = 1$ (No nonlinearity errors.)

**Actual Strain** $\varepsilon = \frac{\varepsilon_I}{4}$ microstrain

where:

$F$ = gage factor of both gage and instrument

$\varepsilon$ = actual strain, in uniaxial tension or compression, in the axial direction

$\varepsilon_I$ = total strain indicated by the strain gage instrument

$\hat{\varepsilon}$ = uncorrected axial strain

**Note 1**: The resistance of each arm in a branch between the power corners of the bridge is nominally the same for both arms.

**Note 2**: A constant-voltage power supply is assumed. Both $\varepsilon$ and $\hat{\varepsilon}$ are expressed in microstrain units.